Learning the eye of the beholder: Statistical modeling and estimation for personalized color perception

Xuanzhou Chen, Austin Xu, Jingyan Wang, Ashwin Pananjady

Contact: xchen920@gatech.edu

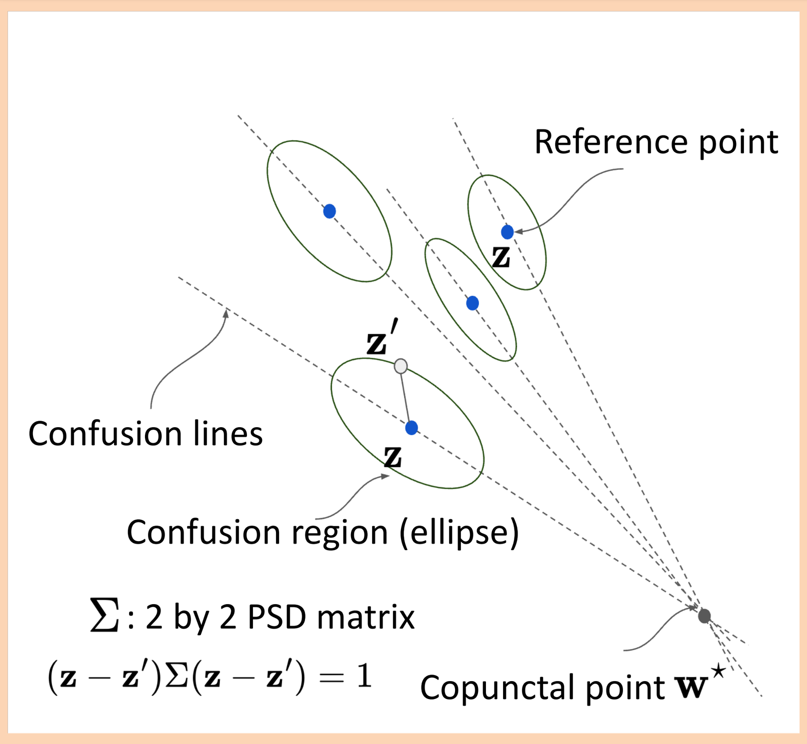
1. Motivation

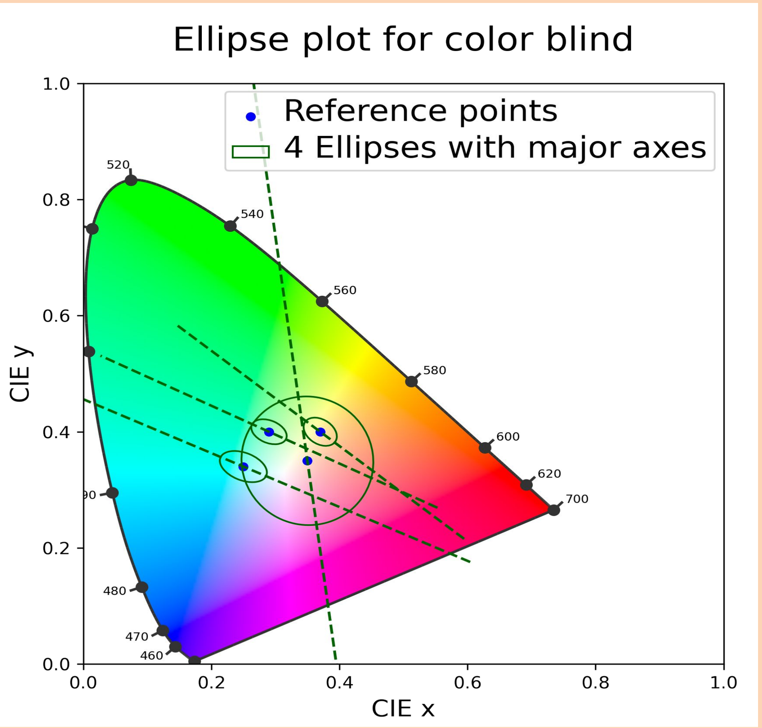
Current color perception models divide people into color normal or color blind.
Need PERSONALIZED color perception diagnosis.

Our contribution

- A unified model on both color blind and color normal
- An algorithm for statistical estimation with theoretical guarantees
- Experimental results from user study

2. Proposed Model

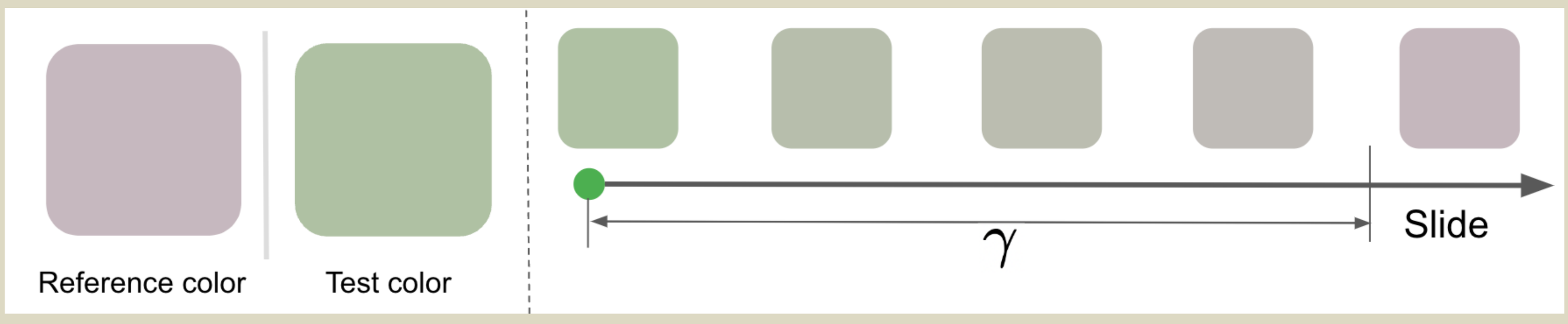




3. PAQ mechanism

- Human perception quantified along a continuous path
- Metric learning (i.e., estimate the PSD matrix Σ)

UI to collect PAQ responses



4. Goal

Given

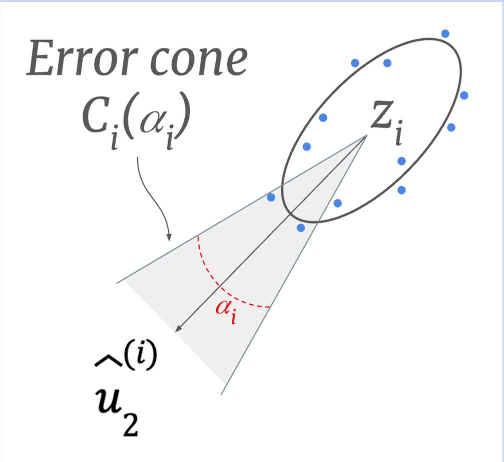
- N as #reference points: $\{\mathbf{z}_i\}_{i=1}^N$
- M as #PAQ responses at each point: $\{\gamma_j\}_{j=1}^M$

Goal: Estimate the user's copunctal point $\mathbf{w}^\star$.

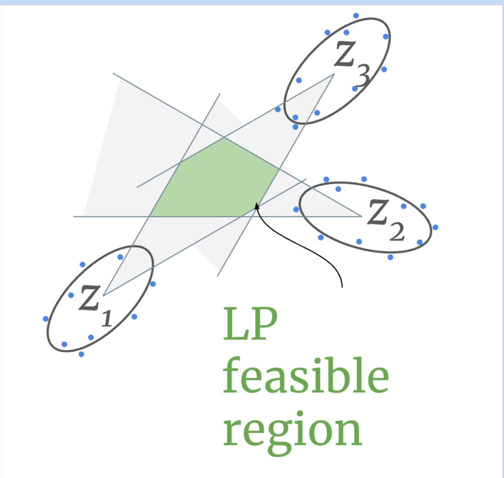
5. Three-step Algorithm

Step 1 [Estimate ellipse]: Given user responses $\{\gamma_j\}_{j=1}^M$, estimate the metric $\mathbf{\Sigma}_i$ at the reference point $\mathbf{z}_i$. $\forall\, i, \widehat{\mathbf{\Sigma}}_i \coloneqq \text{Least\_Squares}(\Sigma_i, \gamma_1, \cdots, \gamma_M)$

Step 2 [Compute major axis]: Given the metric estimate $\widehat{\mathbf{\Sigma}}_i$ at $\mathbf{z}_i$, compute its second eigenvector $\widehat{\boldsymbol{u}}_2^{(i)}$ and construct an error cone $\mathcal{C}_i(\alpha_i)$.



Step 3 [Find copunctal point]: Use Linear Program to estimate the copunctal point by a random point inside N-cone intersection.



6. Theoretical Guarantees

Theorem 1 (Copunctal point estimation error bound)

If $\|\widehat{\mathbf{\Sigma}}_i - \mathbf{\Sigma}_i^*\|_{op} \le \tau_i$ for each $i \in [N]$, then

$$\left\|\widehat{\mathbf{w}} - \mathbf{w}^\star\right\|_2 \lesssim \min_{i,j\in[N]} \|\mathbf{z_i} - \mathbf{z_j}\| \cdot \max\left\{\frac{\tau_i}{|\widehat{\lambda}_1^{(i)} - \widehat{\lambda}_2^{(i)}|}, \frac{\tau_j}{|\widehat{\lambda}_1^{(j)} - \widehat{\lambda}_2^{(j)}|}\right\}.$$

Theorem 2 (Estimation via PAQ)

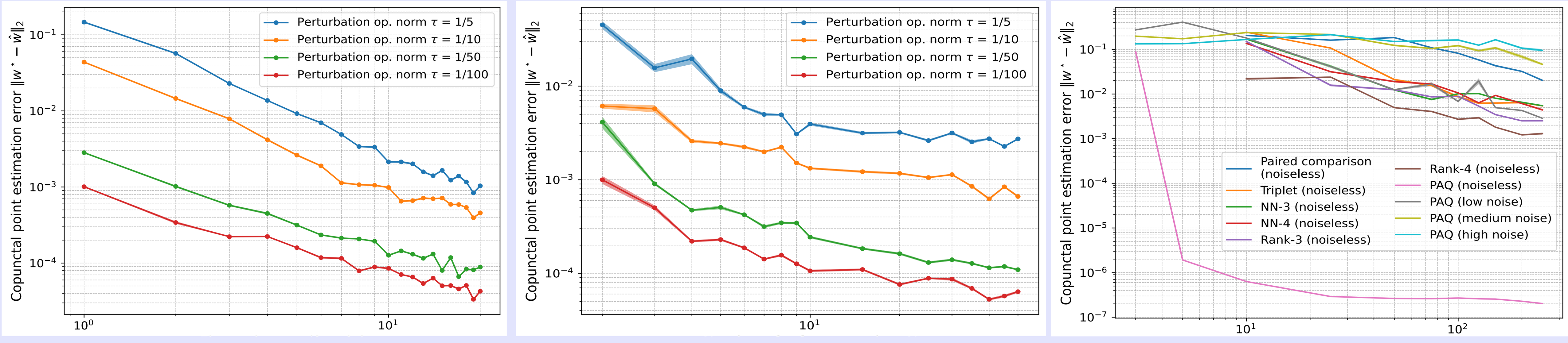
Suppose the number of PAQ measurements M for each reference point satisfies $M \gtrsim \log^3 M$.

Then with high probability,

$$\left\|\widehat{\mathbf{w}} - \mathbf{w}^\star\right\|_2 \lesssim \min_{i,j\in[N]} \|\mathbf{z_i} - \mathbf{z_j}\| \cdot \max\left(\frac{\tau_i}{|\widehat{\lambda}_1^{(i)} - \widehat{\lambda}_2^{(i)}|\sqrt{M_i}}, \frac{\tau_j}{|\widehat{\lambda}_1^{(j)} - \widehat{\lambda}_2^{(j)}|\sqrt{M_j}}\right).$$

7. Numerical Simulations

Copunctal point estimation error vs.



Eigenvalue gap $|\lambda_1 - \lambda_2|$ | Number of reference points N | Number of responses per reference $\overline{M}$

8. User Study

Copunctal point estimation from user PAQ responses: